MEAN–FIELD MORAL HAZARD FOR OPTIMAL ENERGY DEMAND RESPONSE MANAGEMENT

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MOTIVATION & INTUITION

Electric energy cannot easily be stored: supply-demand balance at all times $\Rightarrow$ Act on the supply side?

Issue: Low flexibility (or high cost) production and randomness of renewable energies.

Solution: Demand management, facilitated by the development of smart meters.

How can we encourage demand management and reward it optimally?

In practice. Tariff offers, price signals...

Problem. London Carbon Trial: large variance in the response.

How to improve the responsiveness?

Aïd, Possamaï, and Touzi 2018 - Optimal electricity demand response contracting with responsiveness incentives.

▶ Principal-Agent problem with moral hazard.

The Agent (he) is a risk-averse consumer, who can deviate from his baseline consumption by reducing the mean and the volatility.

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d ds + \int_0^t \sigma(\beta_s) \cdot dW_s, \; t \in [0, T], \tag{1}$$

where W is a d-dimensional Brownian Motion.
A control process for the Agent is a pair $\nu := (\alpha, \beta) \in \mathcal{U}$:

- α is the effort to reduce his consumption in mean;
- β is the effort to reduce the variability of his consumption.

The Principal (she) is a producer (or a retailer) subject to energy generation costs and to consumption volatility costs.

STARTING FROM ONE CONSUMER...

The Principal wants to incentivise the consumer to reduce the mean and the volatility of his consumption.

Moral Hazard: She observes the deviation consumption X of the Agent in continuous time, but not the effort ν he makes.

- She offers him a contract indexed on his deviation consumption:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \Gamma_s \mathrm{d}\langle X\rangle_s + \frac{1}{2}R_A \int_0^T Z_s^2 \mathrm{d}\langle X\rangle_s,$$

for an optimal choice of $\zeta = (Z, \Gamma)$.

Results.

- Optimal contracting allows the system to bear more risk as the resulting volatility may increase;
- The control of the consumption volatility can lead to a significant increase of responsiveness.

The producer is facing a Mean-Field (MF) of correlated consumers and optimise in mean.

Find a way for the Principal to benefit from dealing with this MF of consumers.

She knows the law of the consumption of the pool of consumers.

▶ She can design a new contract in order to penalise / reward a consumer who makes less / more effort than the rest of the pool.

Intuition.

Optimal contracts should consists of two parts:

- A classical part indexed on the deviation consumption of the Agent (previous contract, as in Aïd, Possamaï, and Touzi 2018);
- An additional part indexed on the law of the deviation consumption of others.

Drift and volatility control.

Cvitanić, Possamaï, and Touzi 2018 - Dynamic Programming Approach to Principal–Agent Problems.

Contracting with many Agents.

Élie and Possamaï 2016 - Contracting theory with competitive interacting Agents.

Élie, Mastrolia, and Possamaï 2018 - A tale of a Principal and many many Agents.

Mean–Field Games and Common Noise.

Carmona and Delarue 2018 - Probabilistic theory of mean field games with applications II.

Carmona, Delarue, and Lacker 2016 - Mean Field Games with common noise.

A PRINCIPAL - MF AGENTS PROBLEM

Classical MFG framework: All agents are identical.

▶ Focus on a typical small consumer who has no impact on the global consumption: the representative Agent.

His deviation from his baseline consumption is given by:

$$X_t = x_0 - \int_0^t \alpha_s \mathbf{1}_d ds + \int_0^t \sigma(\beta_s) \cdot dW_s + \int_0^t \sigma^\circ dW_s^\circ, \; t \in [0, T]. \quad (2)$$

where

- W is a d-dimensional idiosyncratic noise;
- W° is a one-dimensional common noise (common random environment as climate hazards).

A control process for the Agent is still a pair $\nu := (\alpha, \beta) \in \mathcal{U}$:

- α is the effort to reduce his consumption in mean;
- β is the effort to reduce the variability of his consumption.

In Aïd, Possamaï, and Touzi 2018, the Principal offers a contract to an Agent indexed on his deviation consumption X.

In the MF case, the Principal faces a Mean Field of Agents and can therefore benefit from this.

She can compute the conditional law of the deviation consumption of other consumers w.r.t the common noise, denoted by $\hat{\mu}$.

$\Rightarrow$ New form of contracts: $\xi(X, \hat{\mu})$.

Optimisation problem of the representative consumer:

$$V_0^A(\xi, \hat{\mu}) := \sup_{\mathbb{P} \in \mathcal{P}} \mathbb{E}^{\mathbb{P}} \left[U_A \left(\xi(X, \hat{\mu}) - \int_0^T \left(c(\nu_t^{\mathbb{P}}) - f(X_t) \right) dt \right) \right], \tag{3}$$

where c is a cost function, f denotes the preference of the Agent toward his deviation consumption, and $U_A(x) = -e^{-R_A x}$.

NEW FORM OF CONTRACTS

Applying the chain rule with common noise in Carmona and Delarue 2018 to the dynamic value function of the Agent, we obtain the following form for the contract:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s, \hat{\alpha}_s^\star, \hat{\mu}_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^t \big(\Gamma_s + R_A Z_s^2\big)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \widehat{\mathbb{E}}^{\hat{\mu}_s}\Big[Z_s^\mu(\widehat{X}_s)\mathrm{d}\widehat{X}_s\Big] + \frac{1}{2}R_A \int_0^T \widehat{\mathbb{E}}^{\hat{\mu}_s}\check{\mathbb{E}}^{\hat{\mu}_s}\Big[Z_s^\mu(\widehat{X}_s)Z_s^\mu(\check{X}_s)\mathrm{d}\langle \widehat{X}, \check{X}\rangle_s\Big]$$
$$+ R_A \int_0^T Z_s \widehat{\mathbb{E}}^{\hat{\mu}_s}\Big[Z_s^\mu(\widehat{X}_s)\mathrm{d}\langle X, \widehat{X}\rangle_s\Big],$$

where

- $\zeta_t = \big(Z_t, Z_t^\mu, \Gamma_t\big)$ is a triple of parameters chosen by the Principal;
- $\hat{\alpha}^\star$ is the optimal drift effort of other consumers;
- $\widehat{X}$ the deviation consumption of others, $\check{X}$ a copy;
- $\widehat{\mathbb{E}}^{\hat{\mu}}$ expectation under $\hat{\mu}$ (w.r.t the common noise).

EQUIVALENCE WITH CONTRACT ON THE COMMON NOISE

What is hidden behind this contract ?

The contract is in fact indexed on:

- X, the deviation consumption of the representative consumer;
- W°, the common noise.

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \big(\Gamma_s + R_A Z_s^2\big)\mathrm{d}\langle X\rangle_s$$
$$+ \int_0^T \sigma^\circ \overline{Z}_s^\mu \mathrm{d}W_s^\circ + \frac{1}{2} R_A \int_0^T \big(\overline{Z}_s^\mu\big)^2 \big(\sigma^\circ\big)^2 \mathrm{d}s + R_A \int_0^T Z_s \overline{Z}_s^\mu \big(\sigma^\circ\big)^2 \mathrm{d}s,$$

where $\overline{Z}_t^\mu := \widehat{\mathbb{E}}^{\hat{\mu}}\big[Z_t^\mu(\widehat{X}_t)\big]$.

▶ If the Principal can offer contract depending directly on the common noise, she can offer this contract, indexed by $\overline{\zeta}_t = \big(Z_t, \overline{Z}_t^\mu, \Gamma_t\big)$.

▶ Contracting on $\hat{\mu}$ or W° leads in fact to the same form of contract.

Given a contract of the previous form,

- optimal effort ν of the representative Agent:

  $$\nu^\star = (\alpha^\star(\mathrm{Z}), \beta^\star(\Gamma)) \;\Rightarrow\; \mathrm{d}X_t = \alpha^\star(\mathrm{Z}) \cdot \mathbf{1}_d \mathrm{d}t + \sigma^\star(\Gamma) \cdot \mathrm{d}W_t + \sigma^\circ \mathrm{d}W_t^\circ,$$

  same as in Aïd, Possamaï, and Touzi 2018 and does not depend neither on Z^μ nor on $\hat{\mu}$;
- MF equilibrium: optimal efforts are the same for all consumers, $\widehat{X} \overset{\mathcal{L}}{\sim} X$ and $\hat{\mu} = \mu^X$;
- from the Principal's point of view, the contract ξ is a function of X and μ^X, the conditional law of X $\Rightarrow$ McKean Vlasov problem.

The Principal wants to minimise, the sum of the conditional expectation of:

- the compensation ξ paid to the consumers;
- the production cost of the consumption deviation, $\int_0^T g(X_t)\mathrm{d}t$;
- the quadratic variation of the deviation consumption, $\int_0^T \mathrm{d}\langle X\rangle_t$;

with respect to the common noise.

Her problem is reduced to a standard control problem:

$$V^P := \sup_{\zeta\in\mathcal{V}} \mathbb{E}\Big[U^P\big(-\mathbb{E}^{\mu_T^L}[L_T]\big)\Big], \quad L_T = \xi_T + \int_0^T g(X_s)\mathrm{d}s + \frac{h}{2}\int_0^T \mathrm{d}\langle X\rangle_s,$$

where μ^L is the conditional law of L and $U^P(c) = -\mathrm{e}^{-R_P c}$ or $U^P(c) = c$.

Two state variables: the conditional law of X (μ^X) and the conditional law of L (μ^L) $\Rightarrow$ HJB technics.

Optimal indexation on the law

$$Z^{\mu,\star} = -Z^\star + \frac{R_P}{R_A + R_P}\overline{u}^P_{\mu^X},$$

leads to the optimal contract:

$$\xi_t = \xi_0 \underbrace{- \int_0^t \mathcal{H}(X_s, \mu_s^X, \zeta_s^\star, \alpha_s^\star)\mathrm{d}s}_{\text{Hamiltonian}} \underbrace{+ \int_0^t Z_s^\star \big(\mathrm{d}X_s - \widetilde{\mathbb{E}}^{\mu_s}[\mathrm{d}\widetilde{X}_s]\big)}_{\text{Penalisation w.r.t the others}}$$

$$\underbrace{+ \frac{1}{2}\int_0^t \Gamma_s^\star \mathrm{d}\langle X\rangle_s}_{\text{Compensation for volatility control}} \quad \underbrace{+ \frac{R_P}{R_A + R_P}\int_0^t \overline{u}^P_{\mu^X}\widetilde{\mathbb{E}}^{\mu_s^X}[\mathrm{d}\widetilde{X}_s]}_{\text{Payment on others}}$$

$$\underbrace{+\frac{1}{2}R_A \int_0^t \left((Z_s^\star)^2 \Big(\mathrm{d}\langle X\rangle_s - (\sigma^\circ)^2 \mathrm{d}s\Big) + \frac{R_P^2}{(R_A + R_P)^2}(\sigma^\circ)^2 (\overline{u}^P_{\mu^X})^2 \mathrm{d}s \right)}_{\text{Compensation for risk due to the risk aversion of the consumer } (R_A)}.$$

Let X° be the deviation consumption without common noise:

$$\mathrm{d}X^\circ_t = -\alpha^\star(Z^\star_t)\mathrm{d}t + \sigma^\star(\Gamma^\star_t)\cdot \mathrm{d}W_t,$$

we can write the contract in term of X° and W°:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}\big(X_s, \zeta^\star_s\big)\mathrm{d}s + \int_0^T Z^\star_s \mathrm{d}X^\circ_s + \frac{1}{2}\int_0^T \big(\Gamma^\star_s + R_A\big(Z^\star_s\big)^2\big)\mathrm{d}\langle X^\circ\rangle_s$$
$$+ \frac{R_P}{R_A + R_P}\sigma^\circ \int_0^T \overline{u}^P_{\mu^X}\mathrm{d}W^\circ_s + \frac{1}{2}\frac{R_A R_P^2}{(R_A+R_P)^2}\big(\sigma^\circ\big)^2 \int_0^T \big(\overline{u}^P_{\mu^X}\big)^2 \mathrm{d}s.$$

Risk–neutral case ($R_P = 0$) $\Rightarrow$ Classical contract for drift and volatility control, indexed on X°, that is the part of the deviation consumption which is really controlled by the Agent.

LINEAR ENERGY VALUE DISCREPANCY

COMPARISON WITH CLASSICAL CONTRACTS

If the energy value discrepancy is linear, i.e. $(f - g)(x) = \delta x$, $x \in \mathbb{R}$:

- the optimal payment rates are deterministic functions of time;
- the optimal $Z^\star$ and $\Gamma^\star$ are the same whether the Principal is risk-averse or risk-neutral;
- the payment $Z^{\mu,\star}$ allows the Principal to choose the risk she wants to bear:

$$Z_t^{\mu,\star} = -Z_t^\star + \frac{R_P}{R_A + R_P}\delta(T - t).$$

We can compare the efforts and the utility of the Principal when she offers contracts indexed by $\zeta^0 = (Z, 0, \Gamma)$:

$$\xi_T = \xi_0 - \int_0^T \mathcal{H}(X_s, \zeta_s^0)\mathrm{d}s + \int_0^T Z_s \mathrm{d}X_s + \frac{1}{2}\int_0^T \left(\Gamma_s + R_A Z_s^2\right) \mathrm{d}\langle X\rangle_s,$$

GAIN IN UTILITY FOR THE PRINCIPAL

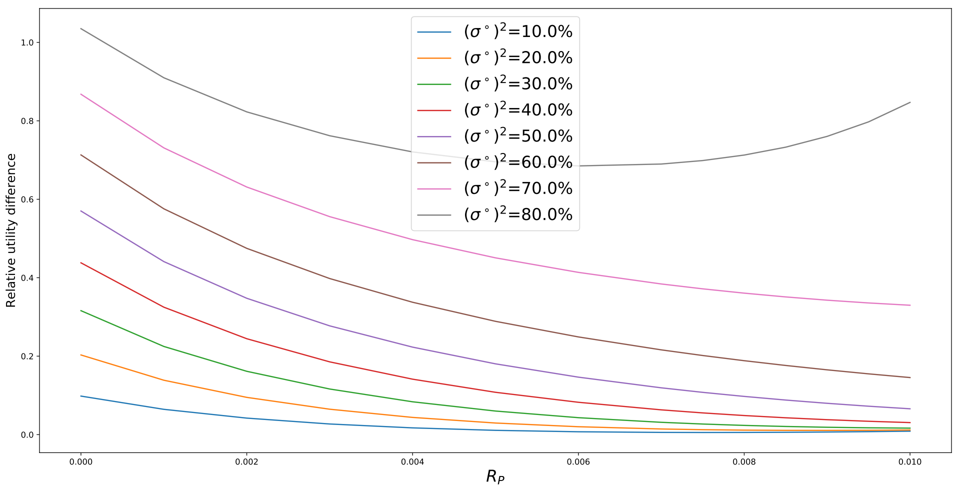

Figure: Relative utility difference.
Variation with respect to R_P and σ°.

EFFORT OF THE AGENTS

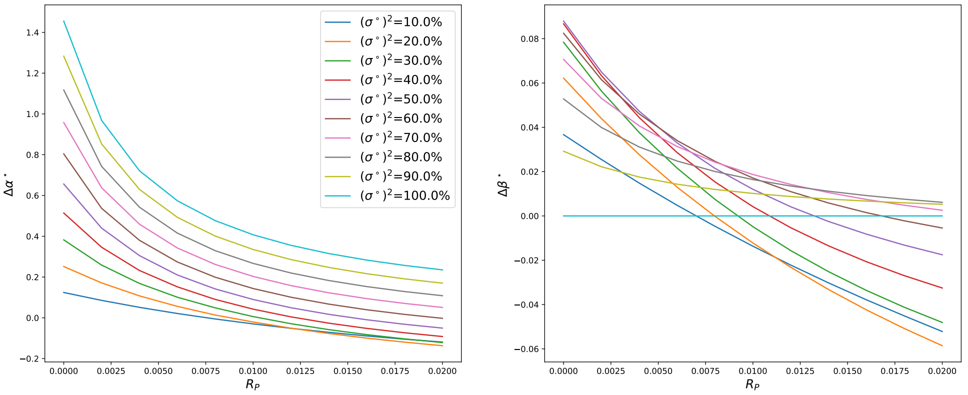

Figure: Relative gain on efforts.
Variation with respect to R_P and σ°.

CONCLUSION

Principal – Mean–Field Agents model, with drift and volatility control, under moral hazard.

- New form of contracts allowing the Principal to benefit from facing a MF of Agents.
- This type of contracts allows her to choose the remaining risk she wants to bear.

At least in the linear energy value discrepancy case,

- there is a gain in utility for the Principal;
- the optimal efforts of the consumers are the same whether the Principal is risk-averse or risk-neutral;
- the consumers make more effort if the risk–aversion parameter of the Principal is small.

Thank you for your attention